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REDUCTION OF STRUCTURAL VIBRATION BY A DYNAMIC ABSORBER

by

J. M. Williams

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SUMMARY (12) 27

This Memorandum describes the transmission of vibration in dynamical systems, and a particular example is studied in detail. This consists of two freely supported elastic plates, connected by a rigid link. It is shown that the addition of a dynamic absorber to such a system can significantly attenuate the transmitted velocities over a chosen narrow band of frequency.

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1 INTRODUCTION

Any dynamic system, excited by a harmonic driving force, will show a greater response at some frequencies than others. At certain frequencies the response velocity is theoretically infinite; such a frequency is called a natural, or resonant, frequency of the system and the system is said to be in a natural mode of oscillation. In practice, due to the structural damping present in such materials as sheet metal, the resonant response peaks remain finite but are still significantly larger than the response at non-resonant frequencies.

In the present work the effect of a dynamic absorber on a resonant system is studied. The dynamic absorber employed is theoretically equivalent to a spring and a mass connected in series; it thus has only one natural frequency, which can be adjusted by changing the mass or the spring stiffness. If an absorber is tuned to a natural frequency of the system under consideration, it can be used to attenuate the response of the system at that frequency. The current investigation is concerned with the effect that such an absorber would have on a system of two connected elastic plates, intended as a rough model of a helicopter gear-box and cabin structure. One of the principal objectives is to study the effectiveness of the absorber over a finite but narrow frequency band containing more than one structural resonance. A mathematical model is set up and some representative results are calculated using methods of numerical analysis.

The system is assumed to be linear, *ie* the magnitude of any velocity in the system is directly proportional to the force producing it. The model used is concerned with the response of the system to a single-frequency harmonic force. More complicated forces can generally be represented as a superposition of such single-frequency inputs because of the assumed linearity.

The actual structure employed consists of two rectangular elastic plates of identical length and width but differing thickness. The thickness of the plates relative to their length is sufficiently small for standard thin plate theory to apply[2]. The plates are freely supported in parallel planes so that their sides are coincident in the x-y plane (Fig 1). A rigid connecting rod is attached to corresponding points on the upper and lower plates ($\underset{\sim}{X}_2$ and $\underset{\sim}{X}_3$) by two pivot joints. Hence the rod is capable of transmitting forces parallel to its axis only and is unable to transmit moments. It is to this connecting rod that the dynamic absorber is attached. Throughout the numerical work presented in this Memorandum the geometry and plate masses are kept constant.

Section 2 describes the mathematical model set up to represent the system and how the various parameters required were chosen. Section 3 contains a discussion of the results obtained by using different dynamic absorbers. Appendices A and B, respectively, provide a more detailed account of the theory of harmonic excitation of thin plates and an explanation of the plate configuration.

2 THEORY

A harmonic excitation is applied to the upper plate at $\underset{\sim}{X}_1$ (Fig 1); the resulting velocity is measured at $\underset{\sim}{X}_4$ on the lower plate. Appendix B explains the exact location of these points. The modal density is defined as the number of natural modes of oscillation in a frequency band of constant width relative to some fixed central frequency (*eg* central frequency ±0.5% for a 1% bandwidth). For this system, modal density increases with frequency, and it is possible to find frequencies such that three or four natural frequencies (of the lower plate) lie within a 1% bandwidth. These resonances are of primary concern in this investigation. The upper plate is twice the thickness of the lower plate and hence has different resonant responses. The modal density of two similar plates is inversely proportional to their thickness; hence on average twice as many resonances are excited within a given frequency band on the lower plate as on the upper plate. Different values of structural damping can be assumed for the model; in practice, a value of the order of 0.01 would often be encountered. It seems reasonable to use a similar value for the damping coefficient of the dynamic absorber.

For a linear system, the response of the system at a point $\underset{\sim}{X}_1$ to an input force at $\underset{\sim}{X}_2$, is governed by equations of the form:-

$$\begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix} = \begin{pmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{pmatrix} \begin{pmatrix} F_x \\ F_y \\ F_z \end{pmatrix} \tag{1}$$

where V_x, V_y, V_z are the components of velocity in the direction of the co-ordinate axes,

F_x, F_y, F_z are the components of the force along the co-ordinate axes,

α_{ij}, $1 \leqslant i, j \leqslant 3$ are constants to be determined.

For a force, $(0,0,F_z)$, acting only in the z-direction, equation (1) becomes:-

$$\begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix} = \begin{pmatrix} \alpha_{13} & F_z \\ \alpha_{23} & F_z \\ \alpha_{33} & F_z \end{pmatrix} .$$

For a thin-plate lying in the (x-y) plane; $V_x \ll V_z$ and $V_y \ll V_z$, (*ie* the greatest response is perpendicular to the plane of the plate). Thus, the approximation, $V_x = V_y = 0$; $V_z = \alpha F_z$ may reasonably be employed. As we are now only considering forces in the z-direction it is convenient to drop the suffix notation. If more than one force is acting on part of the system, equation (1) becomes modified to

$$V = \sum_{i=1}^{n} \alpha_i F_i$$

where n = total number of forces acting.

Fig 2 shows the distribution of forces in the system in schematic form. There are pairs of equal and opposite reactions at the two joints, A and B, and a tension in the spring of the absorber. The damper is assumed to be radially symmetric with the rod on its axis, thus producing no moments. The second diagram shows the resultant velocities corresponding to these forces.

If u = velocity of upper plate at point A ,

$$u = \alpha_1 F_1 + \alpha_2 F_2 \tag{2}$$

where α_1, λ_2 are scalar constants (their evaluation is described in Appendix A),
F_1 = input force,
F_2 = magnitude of reaction at A .

For a harmonic force F_1 , of amplitude $\bar{F}_1$, a complex representation is useful:-

$$F_1 = \bar{F}_1 e^{i\omega t} = \bar{F}_1(\cos \omega t + i \sin \omega t)$$

where ω = angular velocity = $2\pi \times$ frequency.

For a rigid rod, the velocity of the rod and the velocity of the plate at A must be equal. Hence, by Newton's second law

$$F_2 + T - F_3 = m_R \dot{u} \ , \tag{3}$$

where T = force acting on rod due to spring,
F_3 = reaction of rod at B ,
m_R = mass of rod,
$\dot{u}$ = derivative of u with respect to time = acceleration.

Now, $u = \bar{u}e^{i\omega t}$ where $\bar{u}$ = amplitude of u ; whence $\dot{u} = \bar{u}i\omega e^{i\omega t} = i\omega u$.

Hence, (3) becomes:-

$$F_2 + T - F_3 = i\omega m_R u \ .$$

Applying Newton's second law to the absorber yields

$$T = -m_0 \dot{u}_2 = -i\omega m_0 u_2 \tag{4}$$

where T = tension in spring,
m_0 = mass of absorber,
u_2 = velocity of absorber.

But tension in a spring is given by

$$T = ke \ ,$$

where k = stiffness factor of spring,
e = extension of spring.

Differentiating with respect to time gives

$$\dot{T} = k\dot{e} \ .$$

Equation (3) shows that T must have form $\bar{T}e^{i\omega t}$, so that

$$i\omega T = k\dot{e} \qquad (5)$$

but, $\dot{e}$ = rate of change in extension,
= difference in velocity between absorber and rod
= $u_2 - u$.

Hence

$$i\omega T = k(u_2 - u). \qquad (6)$$

Since, also, the velocity of the rod must equal the velocity of the plate at B ,

$$u = \alpha_3 F_3 , \qquad (7)$$

and similarly at the point of response

$$V = \alpha_4 F_3 , \qquad (8)$$

where V = resultant velocity.

In practice, the spring will have some damping factor; when it is so heavily damped that it would just fail to complete a single free oscillation it is said to be critically damped. For a damping coefficient of η_0 times critical damping, equation (4) is replaced by

$$T = - m_0(1 + i\eta_0)i\omega u_2 . \qquad (9)$$

Eliminating F_2, F_3, T, u_2 and u between these equations yields

$$V = \alpha_1\alpha_4F_1\left(i\omega\alpha_2\alpha_3\left[m_R + \frac{m_0}{(1 + i\eta_0) - \omega^2(m_0/R)}\right] + \alpha_2 + \alpha_3\right)^{-1} . \qquad (10)$$

This is a complex velocity, with its amplitude representing the magnitude of V and its argument the phase of V relative to F_1 .

If ω is varied in equation (10) while the other parameters are kept constant, the amplitude of the response (V) will normally be small when

$$\omega = (K/m_0)^{\frac{1}{2}} \qquad (11)$$

since the fraction involving m_0 then reduces to $m_0/i\eta_0$ and η_0 is small. In general this does not correspond to a mathematical minimum of $|V|$ even in the numerical examples of the next section where $m_R = 0$, since α_2 and α_3 are complex, but it is a satisfactory approximation as the results show.

The frequency given by equation (11) is precisely the value of the resonant frequency of the spring-mass system when grounded, *ie* undergoing simple harmonic motion on a fixed base (see Ref 1). Hence, the resonant frequency of the absorber alone is close to an anti-resonant frequency of the complete system where the absorber gives maximum attenuation. The actual value of the velocity will be dependent on the response of the system without the dynamic absorber, the mass of the absorber and its damping coefficient.

3 RESULTS

The response of the system has been computed for a range of input frequency and selected values of the structural damping and the mass of the absorber. The mass of the connecting rod is neglected and the driving frequency is varied over the frequency-band in 20 increments.

It is convenient to define a non-dimensional frequency $\tilde{\omega}_{nm}$ defined by

$$\tilde{\omega}_{nm} = \omega_{nm}\left(\frac{m''\ell_1^4}{B\pi^4}\right)^{\frac{1}{2}} .$$

The significance of the dimensional factor is apparent from the analysis in Appendix A, which also gives the numerical method used and comments on the accuracy. For a thin (4mm) aluminium plate of length 3.2 m and length to width ratio 1.73, $\tilde{\omega}_{nm}$ is numerically about equal to the dimensional frequency ω_{nm} in Hertz ($\omega_{nm} \triangleq 0.9\tilde{\omega}_{nm}$ Hz). The aspect ratio 1.73 was employed because it provided many suitable frequency bands, of which two were finally selected. The plate thicknesses are 4 mm for the lower and 8 mm for the upper.

The two non-dimensional frequency bands used were $\tilde{\omega} = 291.1 \pm 0.5\%$ and $\tilde{\omega} = 63.8 \pm 1.0\%$. The first of these bands included the natural frequencies $\tilde{\omega}_{7,9}$; $\tilde{\omega}_{10,8}$; $\tilde{\omega}_{12,7}$ and $\tilde{\omega}_{19,1}$; the second included $\tilde{\omega}_{4,4}$ and $\tilde{\omega}_{6,3}$.

Initially, the system was considered without a dynamic absorber. Fig 3 shows the response velocity plotted against frequency for the band $\tilde{\omega} = 291.1 \pm 0.5\%$. The quantity V/F is the ratio of the response velocity to the input force (in $m\,s^{-1}\,N^{-1}$). Each curve represents a different value of the structural damping. It can be seen, that for values of structural damping, η, less than 0.0005 all the resonant peaks are clearly discernible. For $\eta > 0.005$, the response is greatly attenuated and the peaks are indistinguishable. This is because the width of the individual peaks is roughly proportional to the damping factor; hence, for large damping, they overlap considerably and form a single peak.

The remaining graphs (Figs 4 to 13) all pertain to a value of 0.005 for the structural damping. The coefficient, η_0 referred to in these figures, is the damping coefficient of the dynamic absorber (tuned to the central frequency of the band).

Figs 4 and 5, respectively, show the average attenuation within the upper and lower bandwidths, plotted against the mass ratio of the absorber. The mass ratio is defined as the mass of the absorber divided by the mass of the lower plate and each curve represents a different damping coefficient. The mean velocity is found by numerically integrating the area under the velocity-frequency graph and dividing by the bandwidth, viz

$$\bar{V} = \frac{1}{0.01\omega_0}\int_{0.995\omega_0}^{1.005\omega_0} V d\omega \quad (\text{for } \tilde{\omega} = \omega_0 \pm 0.5\%) .$$

The average attenuation is defined as

$$20 \log_{10} \frac{\text{mean velocity with damper}}{\text{mean velocity without damper}} = 20 \log_{10} \frac{\bar{V}}{\bar{V}_u} .$$

Not unexpectedly, the attenuation increases with greater absorber mass. Also, as the mass of the absorber tends to zero, the velocity tends to that of the system with no mass attached, that is zero attenuation. For a fixed mass ratio, the attenuation increases as the damping of the absorber is reduced, at least over the range considered.

Figs 6 to 9 (for $\tilde{\omega} = 291.1 \pm 0.5\%$) and Figs 10 to 13 (for $\tilde{\omega} = 63.8 \pm 1\%$) show in more detail how the attenuation varies with different mass ratios, m . Fig 6 shows five curves for mass ratios in the range $2.5 \times 10^{-5} \leqslant m \leqslant 5.0 \times 10^{-4}$, and also the case $m = 0.0$ (*ie* no dynamic absorber); the absorber damping coefficient is 0.01. Fig 10 shows the corresponding result in the second frequency band, three curves only for mass ratios in the range $2.5 \times 10^{-4} \leqslant m \leqslant 1.0 \times 10^{-3}$ are shown; the damping coefficient is also 0.01. Attenuation is again seen to be greatest when the mass of the absorber is highest; in the second frequency band slightly higher masses were needed to produce a chosen degree of attenuation compared with the first band. It should also be noted that the band $\tilde{\omega} = 63.8 \pm 1\%$ displays two distinct peaks, even for a structural damping of 0.005; this is because the peaks are at a greater distance apart than in the higher frequency band. The slightly smaller attenuation can be attributed to the fact that a tuned absorber is most effective over narrow bandwidths.

The set of Figs 7 to 9 may be compared with the set of Figs 11 to 13 to see how the attenuation varies between the upper frequency band and the lower frequency band. In each set the first figure shows the finest tuning, because it refers to the smallest value of added damping, and these two figures in particular show how the attenuation is greatest near the damper resonance in the middle of the band. In general the attenuation reduces away from the middle of the band and will change at some point to an amplification which in turn will increase to a peak at some new resonance of the combined system. One of the purposes of the present study was to check whether positive attenuation could be achieved over the whole of the chosen band, and it can be seen that most, but not all, of the damper arrangements are successful on this point. The worst failure occurs for the smallest mass and damping in the lower frequency band (Fig 11), but even then 10 dB attenuation is available over a bandwidth of nearly 1%.

4 CONCLUSIONS

The present numerical study has been concerned with the use of a tuned damper to suppress the transmission of vibration through a structure over a finite but narrow frequency band. The model chosen was highly simplified but nevertheless possessed many properties that a real structure would have. The input side consisted of a rectangular plate excited by a harmonic force applied at a single point and arranged to have one or two natural resonances within the chosen frequency band. Vibration was transmitted by a light rigid rod to a second thinner plate having twice as many resonances within the band as the input plate. The rod was attached to the input plate at a point remote from the point of application of the input force so that the transmitted force would vary rapidly with frequency as both the input and output plates passed through resonant conditions. The damper was attached to the transmission rod.

It is well-known that tuned dampers can be very effective over a sufficiently narrow frequency band, but there was considerable doubt whether they could cover a band containing three or four structural resonances without being made very heavy. The result of the calculations carried out on the mathematical model used in the present Memorandum is that they can be effective over such a frequency band provided the modal density is high enough for the bandwidth to be only 1 or 2%. Two frequency bands were investigated. The upper had a bandwidth of 1% and one example showed that a mass of 5×10^{-4} times the mass of the responding plate could give about 20-30 dB attenuation over the range. The lower frequency band had a bandwidth of 2% and again one example showed 10-30 dB attenuation for a mass of 10^{-3} times that of the responding plate. Such results are extremely promising and suggest that further analysis associated with experiments on a real structure would be worth undertaking.

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No.	Author	Title, etc
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2	L. Cremer M. Heckl E.E. Ungar	Structure-borne sound. Springer-Verlag (1973)

Appendix A

HARMONIC EXCITATION OF THIN PLATES

A rectangular elastic plate, freely supported at its edges, can undergo oscillations described by the following, fourth order, partial differential equation in ϕ (the bending wave equation, see Ref 2)

$$B\nabla^4\phi(x,y) - \omega^2 m''\phi(x,y) = 0 \,, \tag{A-1}$$

where B = flexual rigidity of plate,

ω = eigen frequency

$$\nabla^2 f(x,y,z) \equiv \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} \quad \text{in cartesian co-ordinates,}$$

$$m'' = \text{surface mass density of plate} = \frac{\text{mass of plate}}{\text{area of one face}}$$

and $\phi(x,y)$ is proportional to the normal displacement of the plate at a point (x,y); the undeformed plate lying in the plane $z = 0$.

For a rectangular plate $x \in [0,\ell_1]$, $y \in [0,\ell_2]$, ϕ must also satisfy the following boundary conditions, as there is no displacement along the plate edges

$$\phi(x,0) = \phi(0,y) = \phi(x,\ell_2) = \phi(\ell_1,y) = 0 \,. \tag{A-2}$$

A family of solutions to (A-1) and (A-2) can be found, having the form

$$\phi_{nm}(x,y) = \sin\left(\frac{n\pi x}{\ell_1}\right) \sin\left(\frac{m\pi y}{\ell_2}\right) \quad n,m \text{ integers} \tag{A-3}$$

where the suffix nm is used to distinguish between the different solutions ϕ. The corresponding values for the eigen frequencies ω_{nm} (which are in fact the natural frequencies associated with the (n,m)-th mode) are

$$\begin{aligned}\omega_{nm} &= \left(\frac{B}{m''}\right)^{\frac{1}{2}} \left[\left(\frac{n\pi}{\ell_1}\right)^2 + \left(\frac{m\pi}{\ell}\right)^2\right] \\ &= \left(\frac{B\pi^4}{m''\ell_1^4}\right)^{\frac{1}{2}} \left[n^2 + m^2\left(\frac{\ell_1}{\ell_2}\right)^2\right] \,.\end{aligned} \tag{A-4}$$

For an input force distribution $F(x,y)$ with frequency ω, it can be shown that the velocity at a point x_1,y_1 on the plate is given by:-

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$$V(x_1,y_1) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \frac{4\phi_{nm}(x_1,y_1)}{\ell_1\ell_2 m''(\omega_{nm}^2 - \omega^2)} \int_0^{\ell_2}\int_0^{\ell_1} i\omega F(x,y)\phi_{nm}(x,y)\,dxdy \ . \qquad \text{(A-5)}$$

For a point force magnitude F, at (x_0,y_0), $F(x,y) = F\delta(x - x_0)\delta(y - y_0)$, where δ is the Dirac delta-function.

Hence

$$\int_0^{\ell_2}\int_0^{\ell_1} F(x,y)\phi_{nm}(x,y)\,dxdy = \int_0^{\ell_2}\int_0^{\ell_1} F\delta(x - x_0)\delta(y - y_0)\sin\left(\frac{n\pi x}{\ell_1}\right)\sin\left(\frac{m\pi y}{\ell_2}\right) dxdy$$

$$= F\sin\left(\frac{n\pi x_0}{\ell_1}\right)\sin\left(\frac{m\pi y_0}{\ell_2}\right) \ . \qquad \text{(A-6)}$$

Hence (A-5) becomes

$$V(x_1,y_1) = \frac{4i\omega F}{\ell_1\ell_2 m''}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \frac{\sin\left(\frac{n\pi x_0}{\ell_1}\right)\sin\left(\frac{n\pi x_1}{\ell_1}\right)\sin\left(\frac{m\pi y_0}{\ell_2}\right)\sin\left(\frac{m\pi y_1}{\ell_2}\right)}{\omega_{nm}^2 - \omega^2} \ . \qquad \text{(A-7)}$$

If structural damping η is present, the term ω_{nm}^2 is replaced by the complex term, $\omega_{nm}^2(1 + i\eta)$ and if $V(x_1,y_1) = \alpha F$ then

$$\alpha = \frac{4i\omega}{m''\ell_1\ell_2}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \frac{\sin\left(\frac{n\pi x_0}{\ell_1}\right)\sin\left(\frac{n\pi x_1}{\ell_1}\right)\sin\left(\frac{m\pi y_0}{\ell_2}\right)\sin\left(\frac{m\pi y_1}{\ell_2}\right)}{\omega_{nm}^2(1 + i\eta) - \omega^2} \ . \qquad \text{(A-8)}$$

Thus, the total response is equal to the sum of the responses in each individual mode; the modes do in fact form an orthogonal basis from which any response distribution may be constructed in a unique manner. In general, at a given frequency ω_0 say an infinite set of natural modes will be simultaneously excited; if ω_0 is the eigen frequency for a particular mode, that mode will be excited the most.

The numerical calculations were carried out on the ICL 1906S computer at RAE. In practice, the infinite series have to be truncated after a finite number of terms, and this number was chosen to be 30 for both n and m; equation (A-8) was, therefore, truncated after 900 terms. These limits on n and m were decided after accuracy tests and may be compared with the modal numbers of the resonances that occur within the frequency band of the input force, viz $(n,m) = (7,9)$, $(10,8)$, $(12,7)$, $(17,1)$ for $\bar{\omega} = 291.1 \pm 0.5\%$ and $(4,4)$, $(6,3)$ for $\bar{\omega} = 63.8 \pm 1.0\%$. In general, it was found necessary to pay considerable attention to the accuracy of the calculations and single-precision arithmetic was found to be insufficient. Accordingly, the algebra was rewritten in terms of real variables and the program converted to double-precision; it was then possible to meet all the accuracy tests that were tried.

Appendix B

PLATE CONFIGURATION

Consider a point $(x_0, y_0) = \left((\ell_1/p), (\ell_2/q)\right)$ for integer p,q . Then $\sin\left((n\pi/\ell_1)x_0\right) \sin\left((m\pi y_0/\ell_2)\right) = 0$ whenever p is a factor of n , or q a factor of m . This gives the condition *for zero response of the plate* ($\phi_{nm} = 0$) under excitation at the point (x_0, y_0), *ie* excitation at a node in a particular mode produces no response in that mode. For excitation within a narrow bandwidth, containing only a few natural modes, the greater part of the response is contributed by the resonance, or near-resonance, of these modes, with only a small contribution from modes outside this frequency band. It is, therefore, necessary to ensure that the excitation and response points are not at node points for these modes.

Consider a set of natural modes within a frequency band. Let N be the maximum value of n for these nodes, and M be the corresponding maximum of m . Let $p = 2N$ and $q = 2M$. This ensures that p will not be a factor of n for modes under consideration, nor q of m .

Let

$$(x_0, y_0) = \left(\frac{\ell_1}{p}, \frac{\ell_2}{q}\right) = \left(\frac{\ell_1}{2N}, \frac{\ell_2}{2M}\right) ;$$

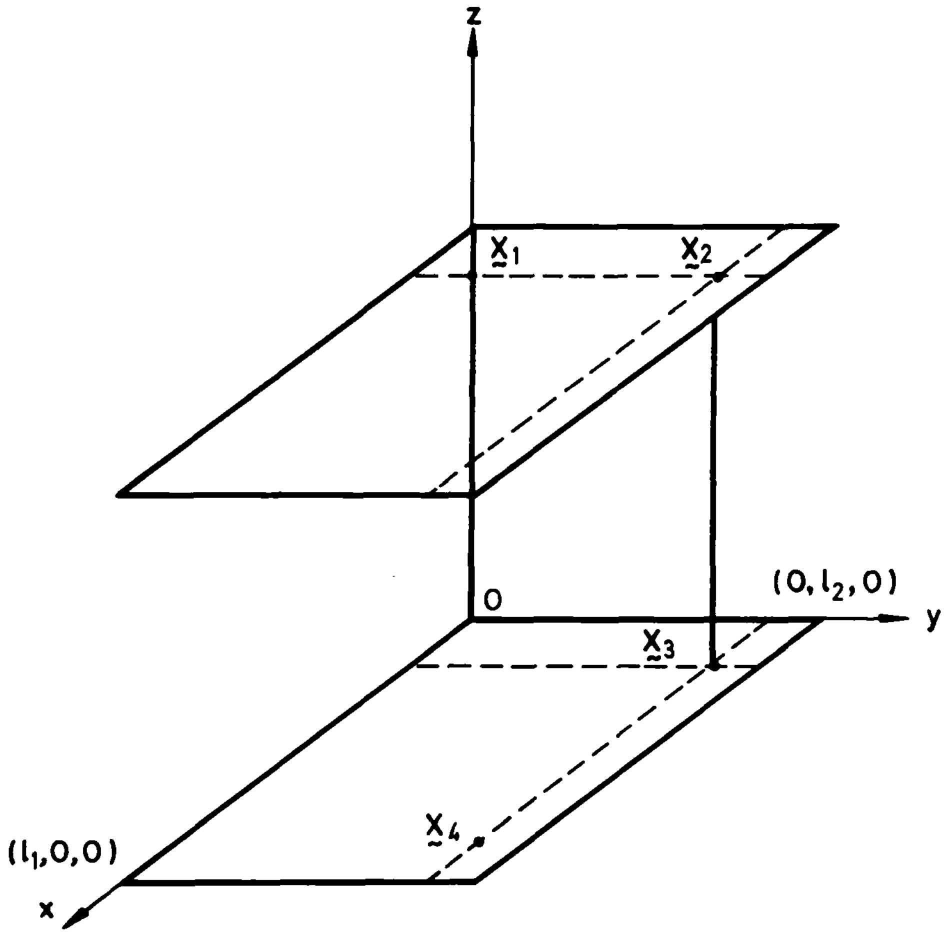
then (x_0, y_0) is not a node for the natural modes in this frequency band and by symmetry neither are $(x_0, \ell_2 - y_0)$ nor $(\ell_1 - x_0, \ell_2 - x_0)$. The configuration shown in Fig 1 makes use of these points for $\underset{\sim}{X}_1, \underset{\sim}{X}_2 [= \underset{\sim}{X}_3]$, and $\underset{\sim}{X}_4$. Provided the values M and N used in deriving (x_0, y_0) relate to the more flexible plate (since the thinner plate possesses higher modes for a fixed frequency band), none of these points can be a node in any of the modes of interest, for either plate.

LIST OF SYMBOLS

B	flexural rigidity of plate = $Et_0^3/12$ where E is Young's modulus and t_0 is the plate thickness	
F_i	force acting at point i	
T	tension in absorber spring	
ℓ_1	plate length	see Fig 1
ℓ_2	plate width	see Fig 1
m	ratio of absorber mass to lower-plate mass	
m_R	mass of rod	see Fig 2
m_0	mass of absorber	see Fig 2
m''	plate mass per unit area	
t	time	
u	vibration velocity	see Fig 2
u_2	vibration velocity of absorber mass	see Fig 2
v	vibration velocity of reference point	see Fig 2
$(x,y,z) \equiv \underset{\sim}{X}$	cartesian coordinates	
$\underset{\sim}{X}_1$	point of application of exciting force	see Fig 1
$\underset{\sim}{X}_2$	point of attachment of transmitting rod on upper (input) plate	see Fig 1
$\underset{\sim}{X}_3$	point of attachment of transmitting rod on lower (responding) plate	see Fig 1
$\underset{\sim}{X}_4$	reference measuring point	see Fig 1
z_0	plate separation distance	
[illegible]$_i$	velocity of some specified point per unit force applied at point i	
[illegible]	structural damping coefficient	
[illegible]$_0$	damping coefficient of the absorber	
[illegible]	non-dimensional displacement of the plate	
[illegible]	radian frequency	
[illegible]	non-dimensional frequency $\equiv \omega\left(m''\ell_1^4/B\pi^4\right)^{\frac{1}{2}}$	

A single numerical suffix refers to points 1 to 4 (see Fig 1).

A double suffix of the type nm refers to mode (n,m) of the plate, see equation (A-3)



$\underset{\sim}{X}_1 = (x_0 , y_0 , z_0)$

$\underset{\sim}{X}_2 = (x_0 , l_2 - y_0 , z_0)$

$\underset{\sim}{X}_3 = (x_0 , l_2 - y_0 , 0)$

$\underset{\sim}{X}_4 = (l_1 - x_0 , l_2 - y_0 , 0)$

Fig 1 System configuration

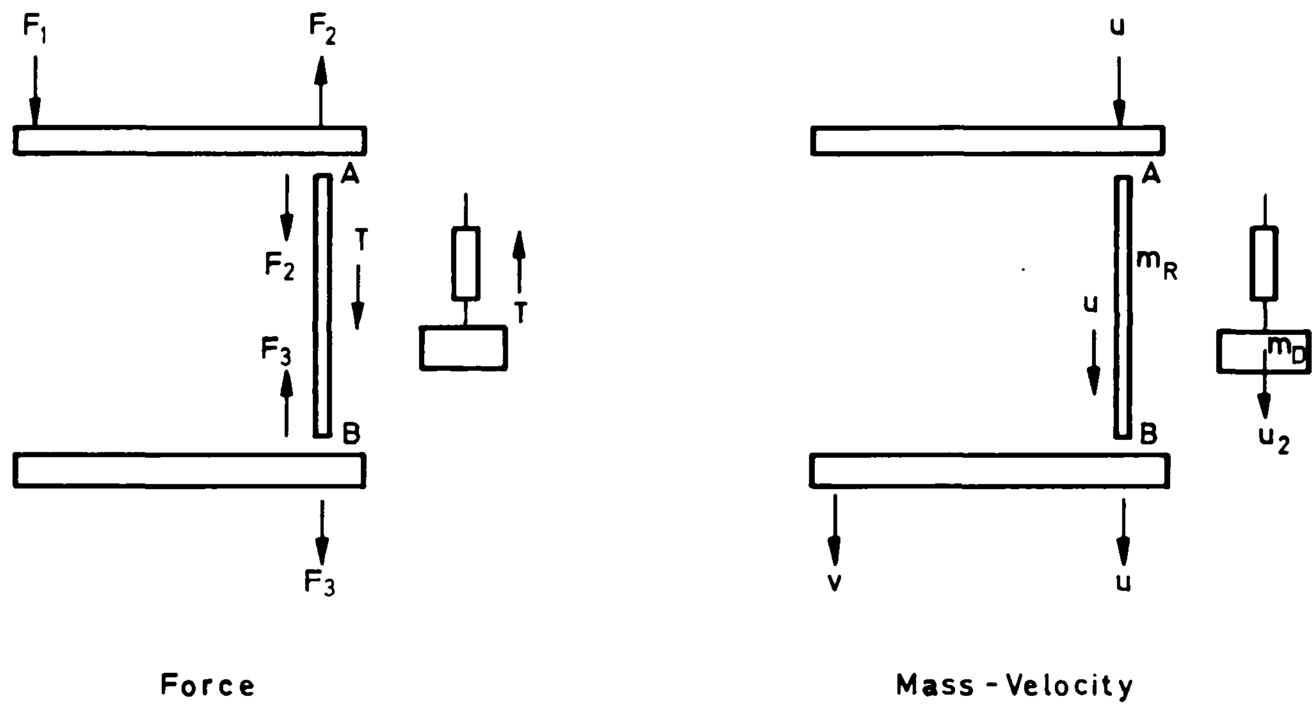


Fig 2 Sketch showing forces and velocites

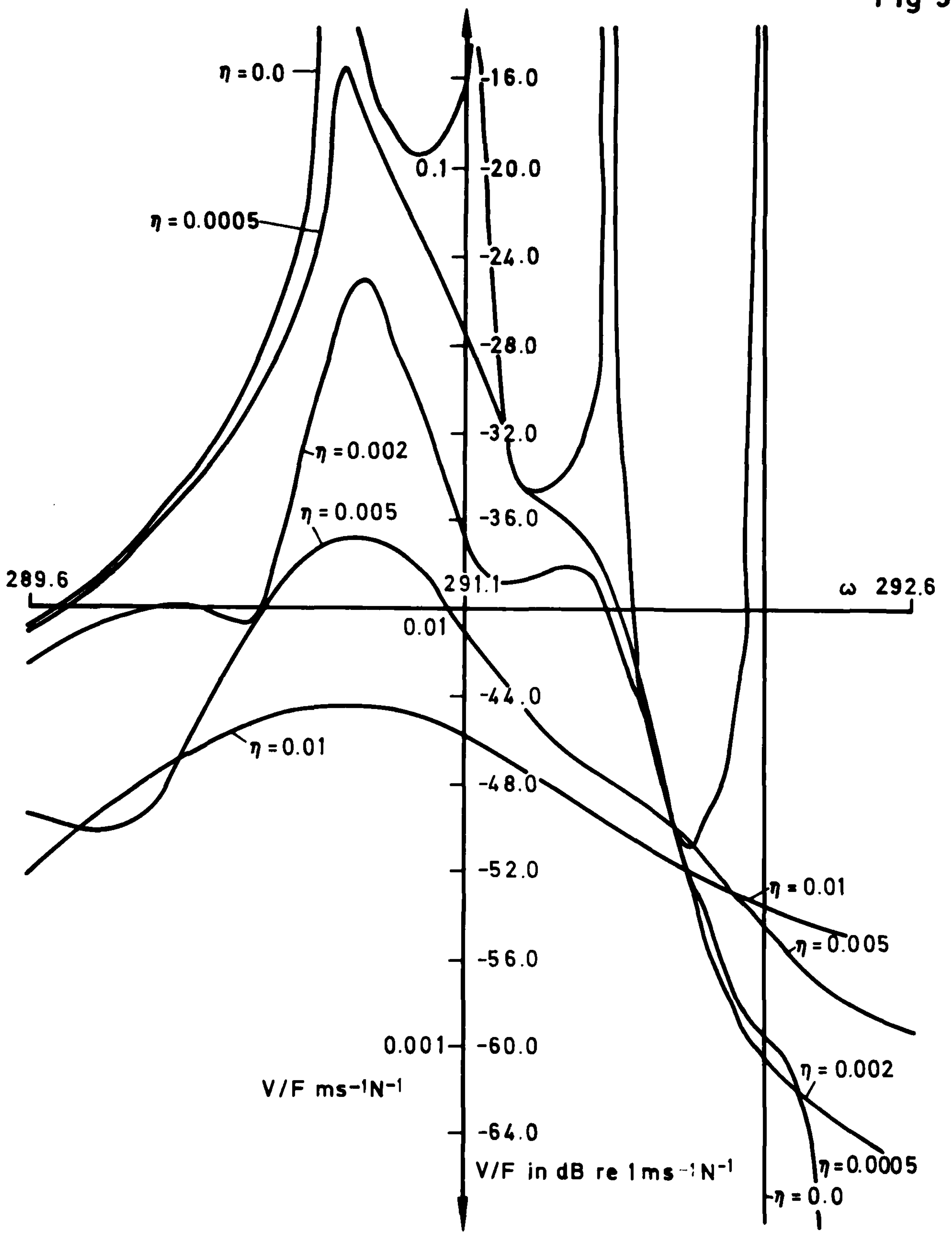


Fig 3 Effect of structural damping on response

Fig 4

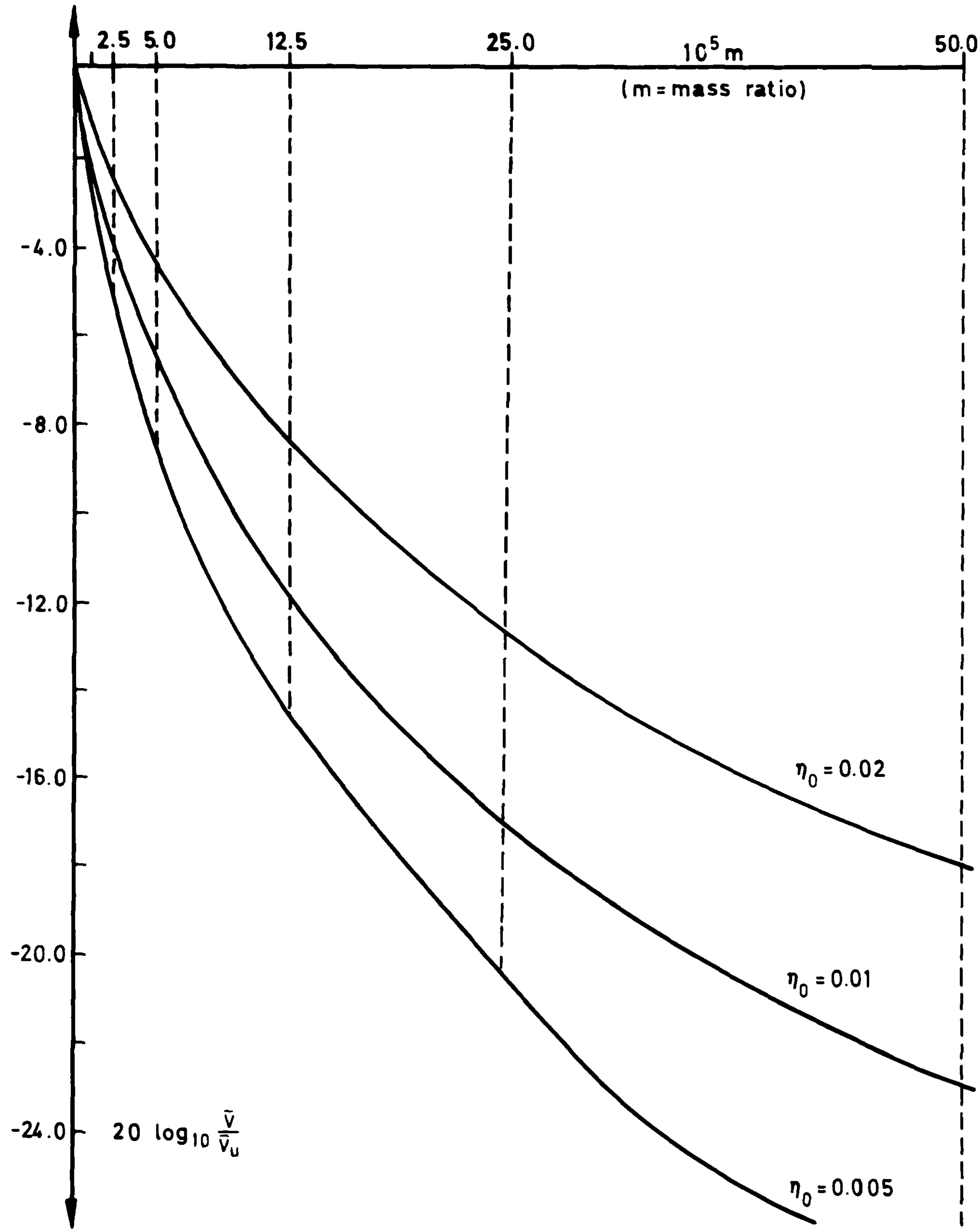


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Fig 4 Average attenuation in bandwidth
$\tilde{\omega} = 291.1 \pm 0.5\,\%$

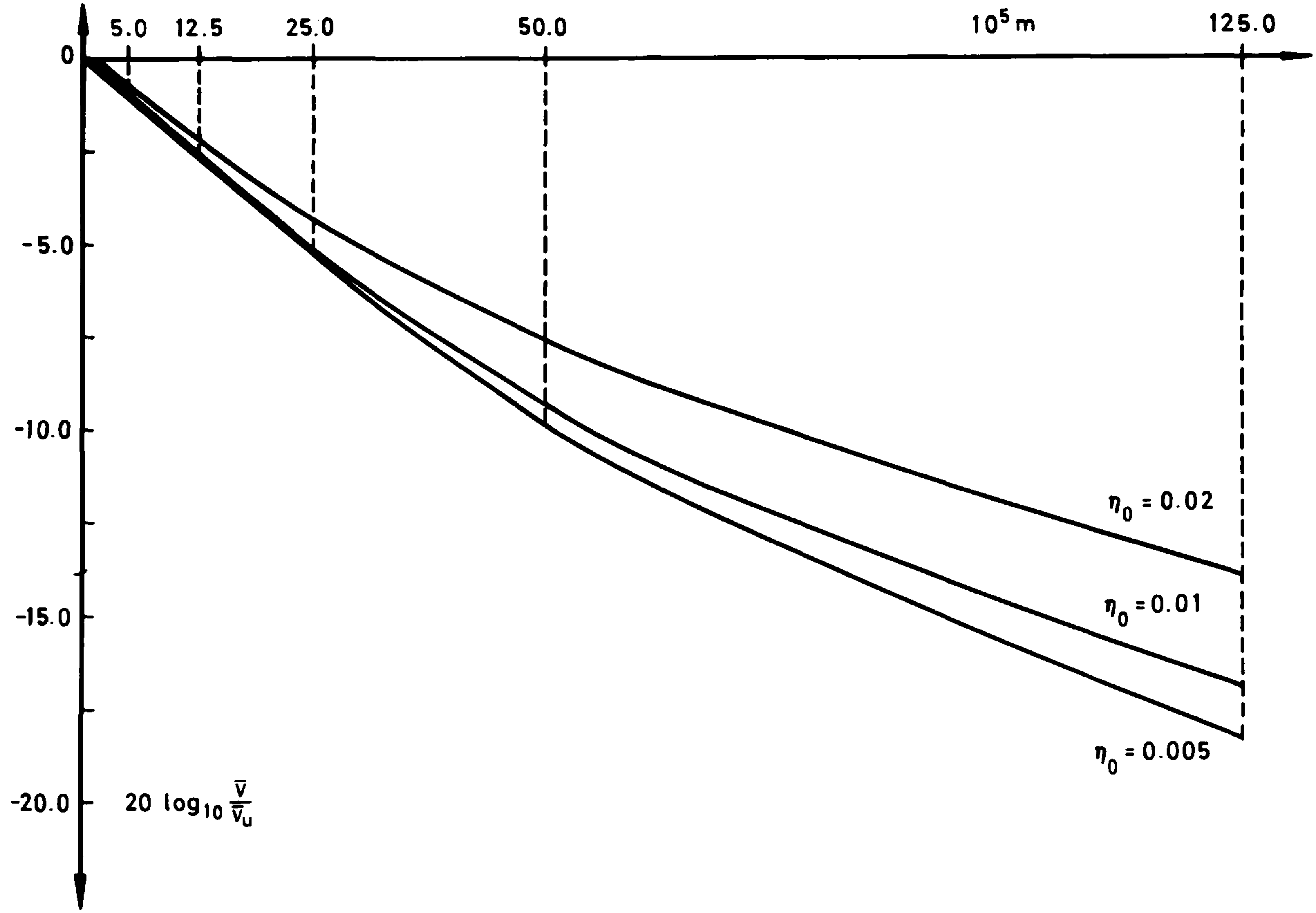

Fig 5 Average attenuation in bandwidth $\tilde{\omega} = 63.8 \pm 1.0\,\%$

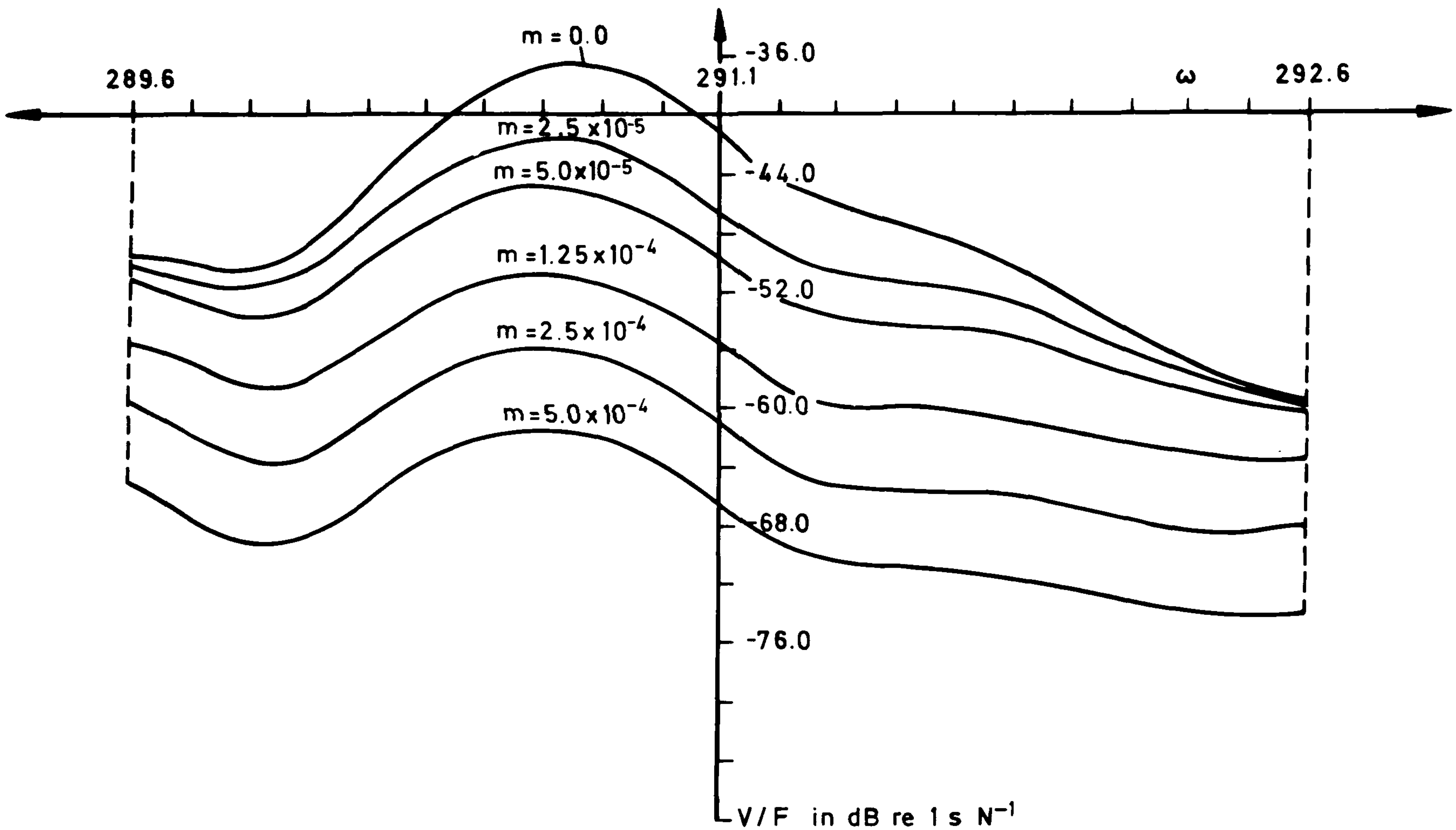

Fig 6 Response for different masses $\eta_o = 0.01$

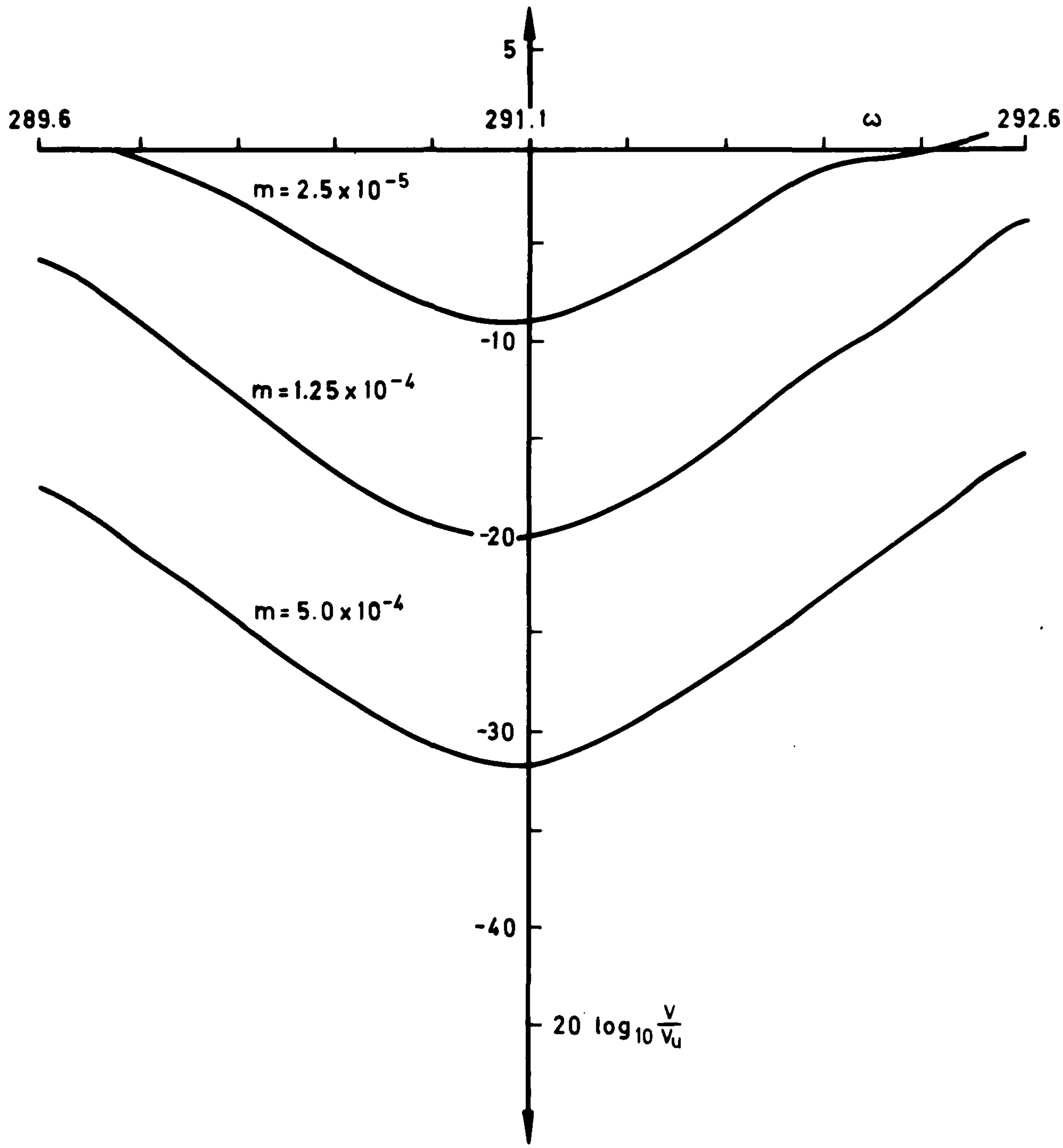

Fig 7 Velocity attenuation resulting from the absorber over a 1% frequency band. $\eta_0 = 0.005$

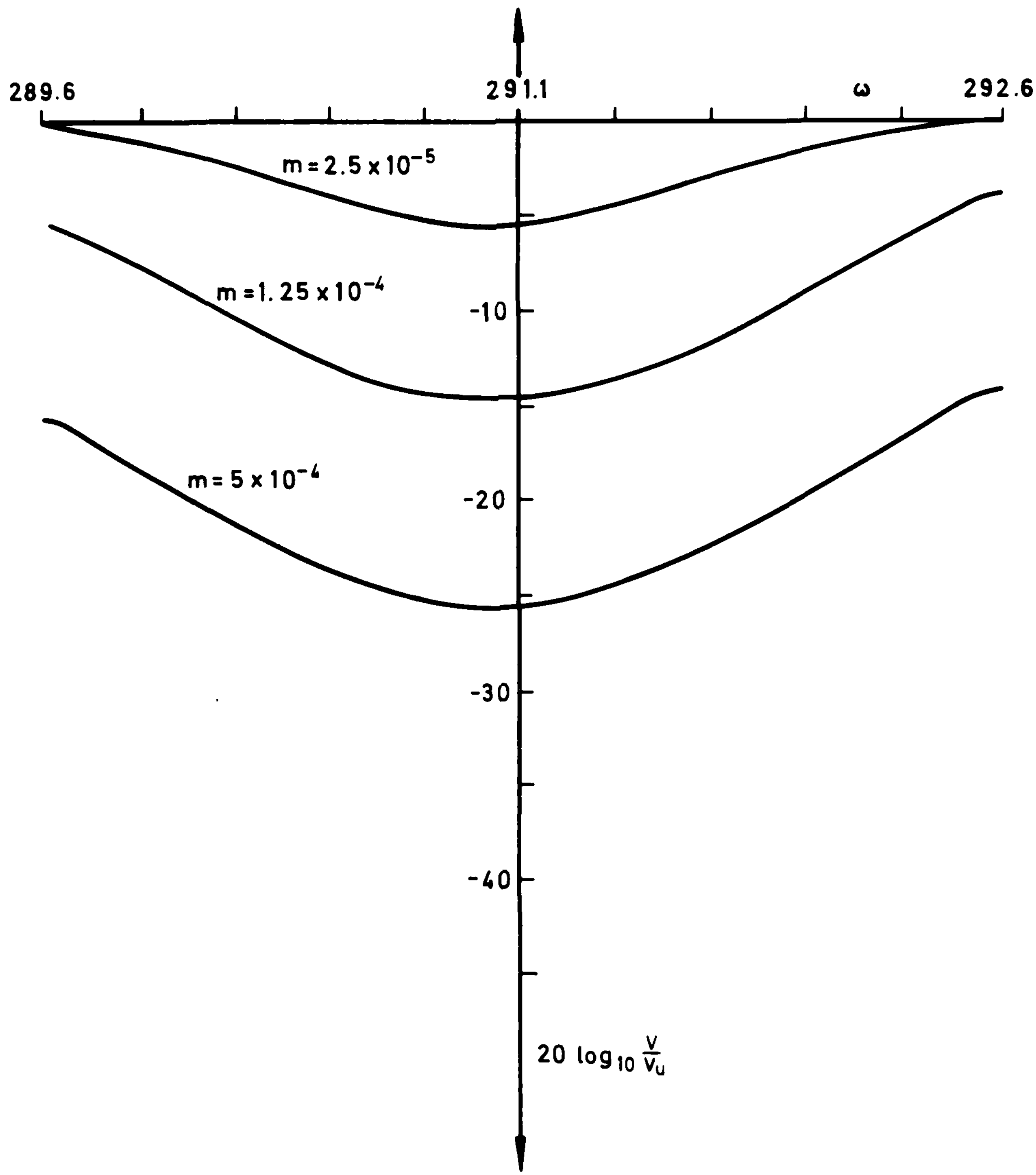

Fig 8 Velocity attenuation resulting from the absorber over a 1% frequency band $\eta_0 = 0.01$

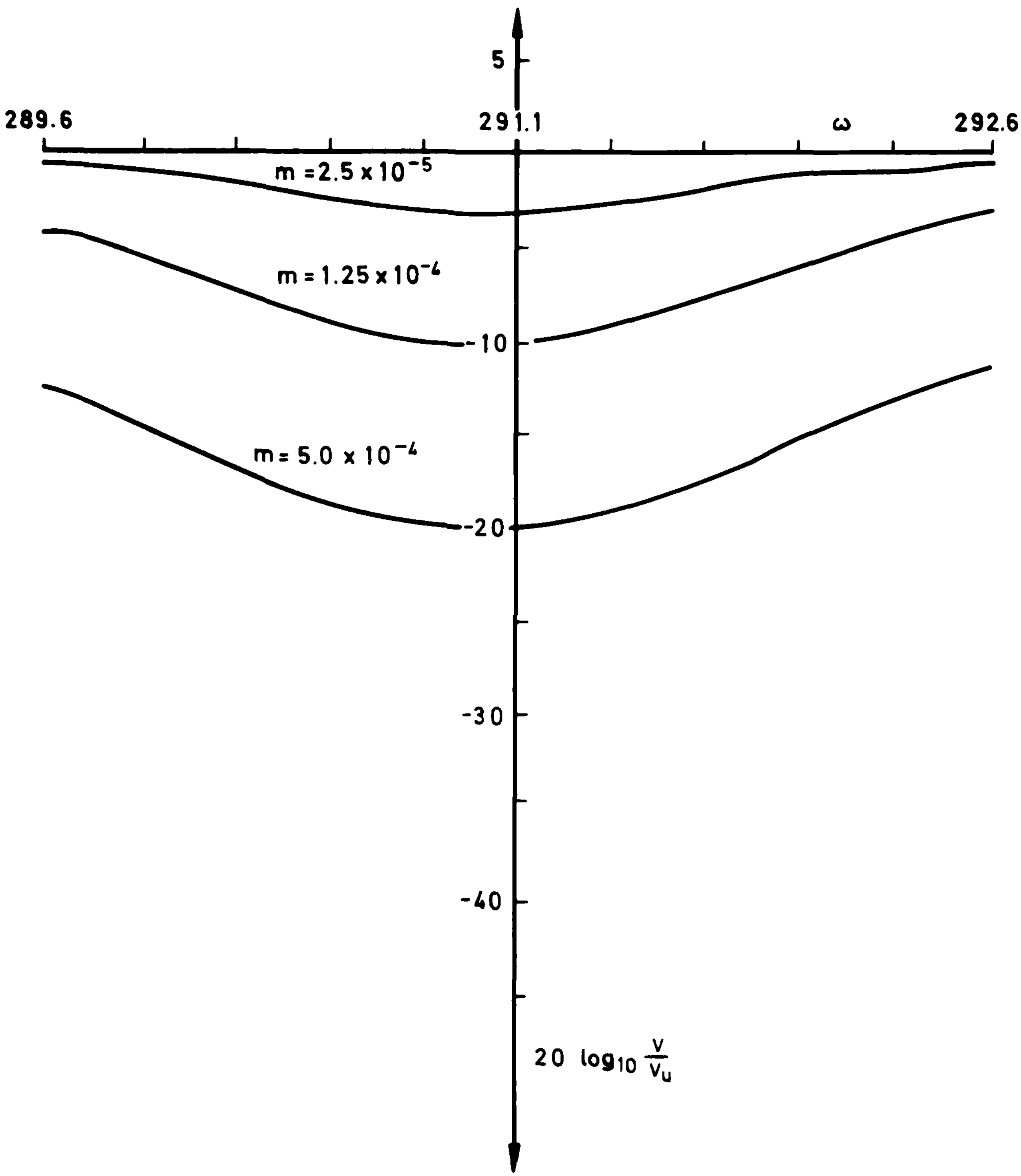

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Fig 9 Velocity attenuation resulting from the absorber over a 1% frequency band $\eta_0 = 0.02$

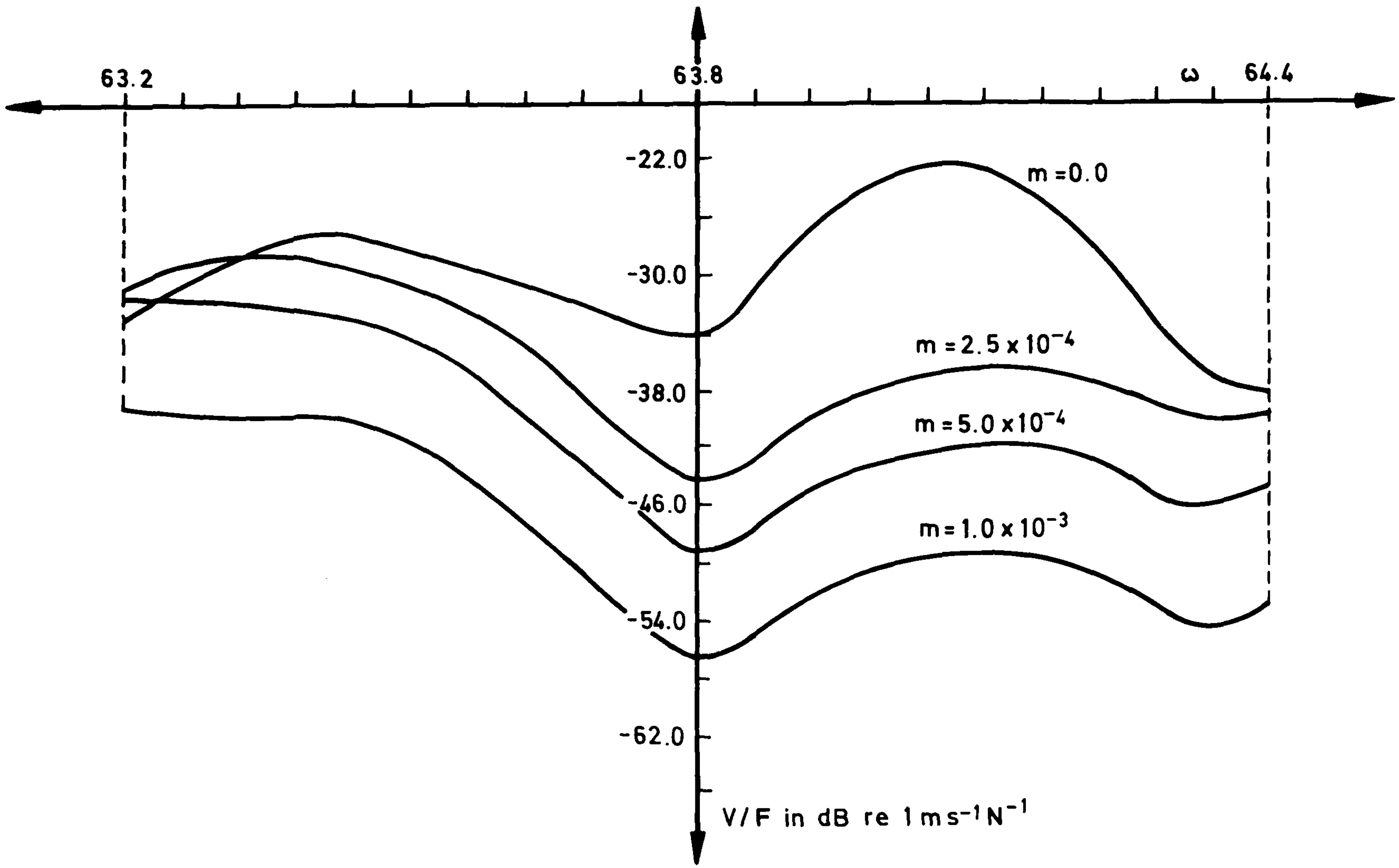

Fig 10 Response for different masses. Damping coefficient = 0.01

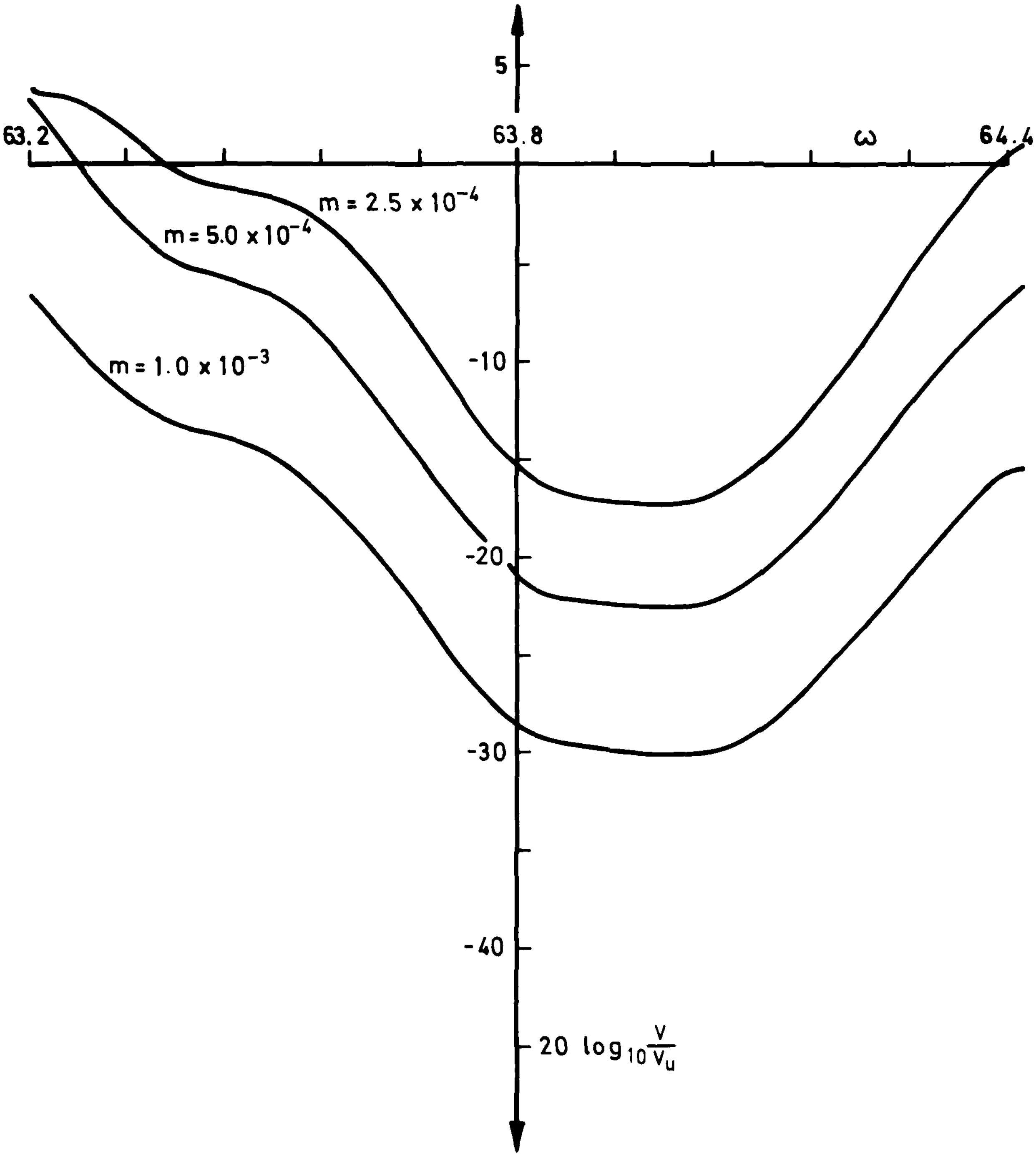

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Fig 11 Velocity attenuation resulting from the absorber over a 2% frequency band $\eta_0 = 0.005$

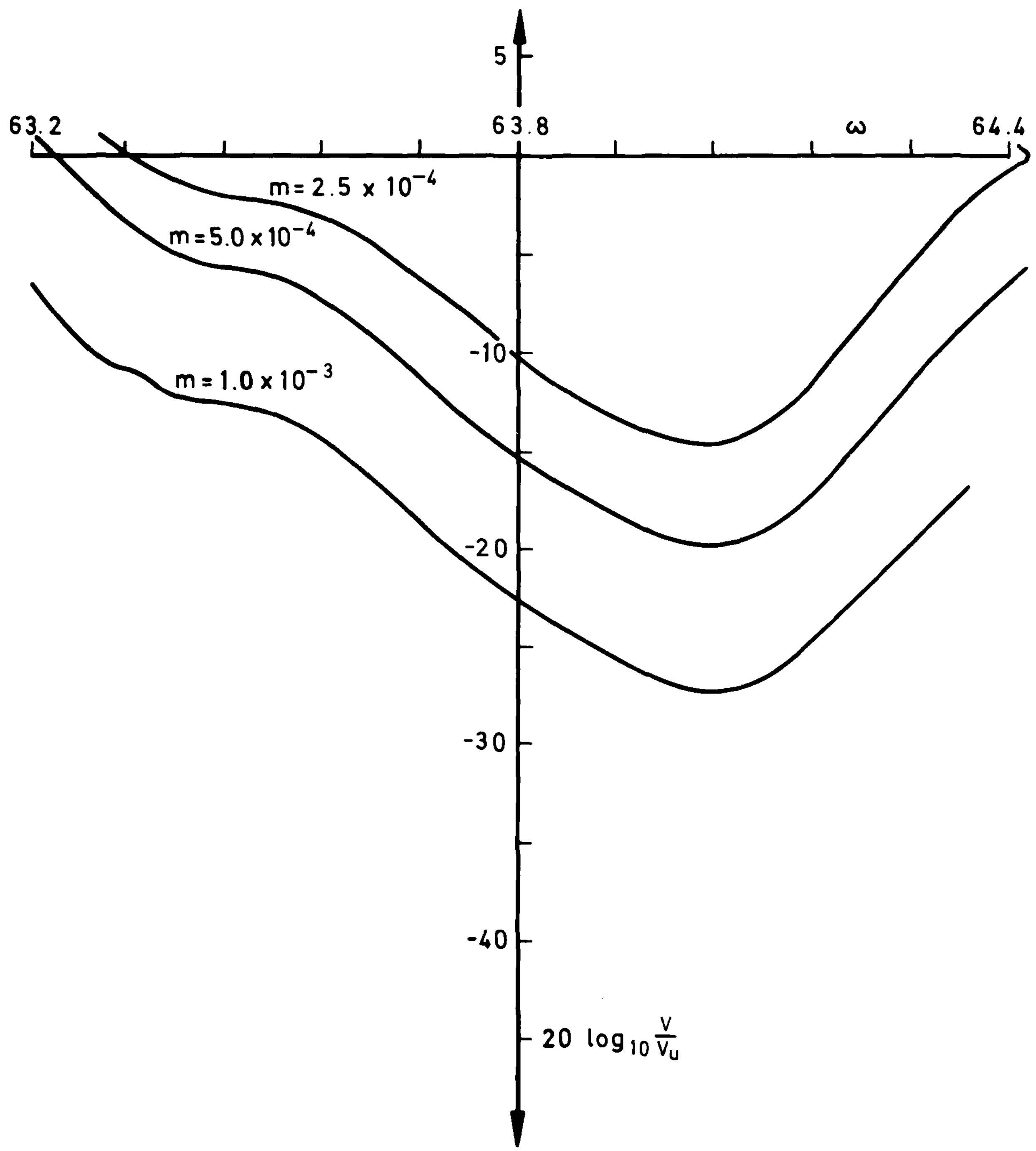

Fig 12 Velocity attenuation resulting from the absorber over a 2% frequency band $\eta_0 = 0.01$

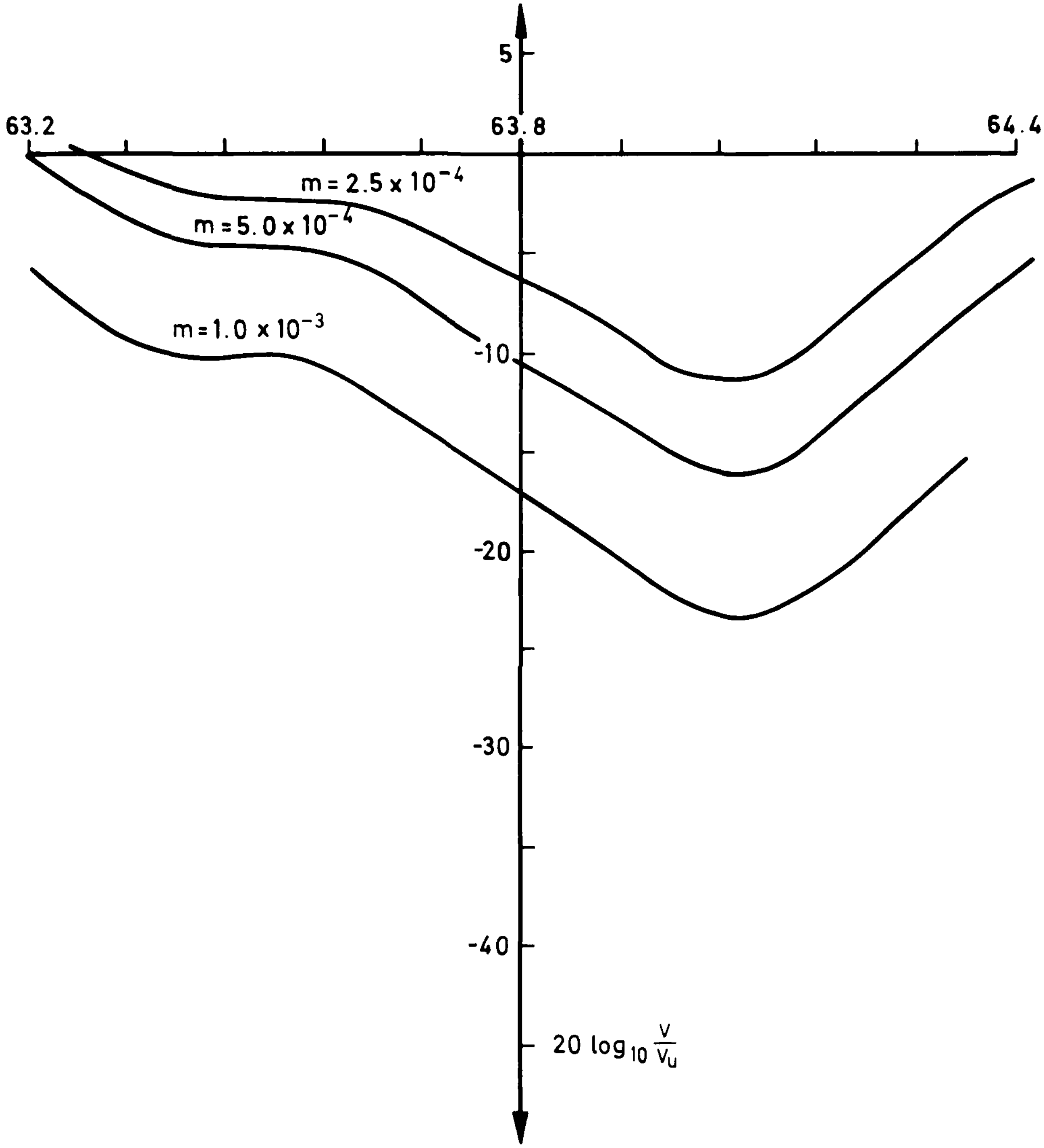

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Fig 13 Velocity attenuation resulting from the absorber over a 2% frequency band $\eta_0 = 0.02$

REPORT DOCUMENTATION PAGE

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17. Abstract

This Memorandum describes the transmission of vibration in dynamical systems and a particular example is studied in detail. This consists of two freely supported elastic plates, connected by a rigid link. It is shown that the addition of a dynamic absorber to such a system can significantly attenuate the transmitted velocities over a chosen narrow band of frequency.

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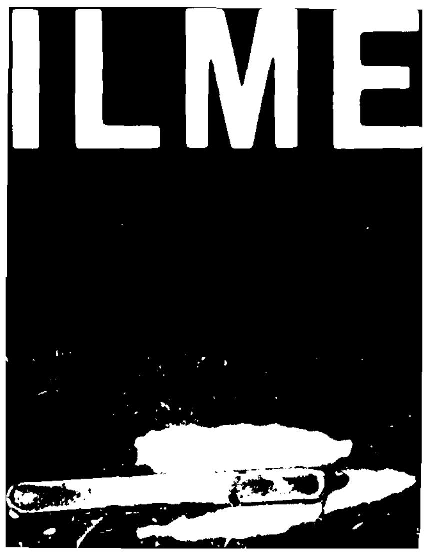
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